GRADE 12 EXAMINATION
NOVEMBER 2016

ADVANCED PROGRAMME MATHEMATICS: PAPER I
MODULE 1: CALCULUS AND ALGEBRA

MARKING GUIDELINES

Time: 2 hours 200 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

1.1 (a) $x > -3$ $x + 3 + 2x = 4$ $\quad$ $x < -3$ $-x - 3 + 2x = 4$

$\therefore 3x = 1$ $\quad$ $x = 7$

$\therefore x = \frac{1}{3}$ $\quad$ n/a (7)

(b) $\frac{x^2}{8} = \frac{1}{2}$

$x = \pm 2$ (4)

(c) $2lnx - \frac{3}{ln\,x} = 1$ $\quad$ $k = lnx$

$2k^2 - k - 3 = 0$

$k = 1{,}5$ $\quad$ $k = -1$

$x = e^{1{,}5}$ $\quad$ $x = e^{-1}$ (8)

1.2 Domain: $e^2 - x^2 > 0$ $\quad$ i.e. $-e < x < e$

Range: max value = $lne^2 = 2$ $\quad$ $y \leq 2$ (8)

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QUESTION 2

2.1 $i^4 = 1$

$\sqrt{1} = 1$ (2)

2.2 $(a + 3i)(3 + 2i) = bi$

$3a + 2ai + 9i - 6 = bi$

Compare real: $3a - 6 = 0$

$a = 2$

Compare imaginary: $2a + 9 = b$

$b = 13$ (7)

2.3 $x = q - \sqrt{3}i$ is also a root

Sum of roots = $2q = 2$

$q = 1$

Product of roots = $q^2 + 3 = p$

$p = 4$.

OR

$x - q = -\sqrt{3}i$

$x^2 - 2qx + q^2 + 3 = 0$

$\therefore q = 1 \quad p = 4$ (7)

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QUESTION 3

3.1 (a) True
(b) False
(c) False
(d) False
(e) True
(f) True (12)

3.2 Continuity:

$$\lim_{x \to 2^-} \left(p - x^2\right) = \lim_{x \to 2^+} \left(qx + 10\right)$$

$$p - 4 = 2q + 10$$

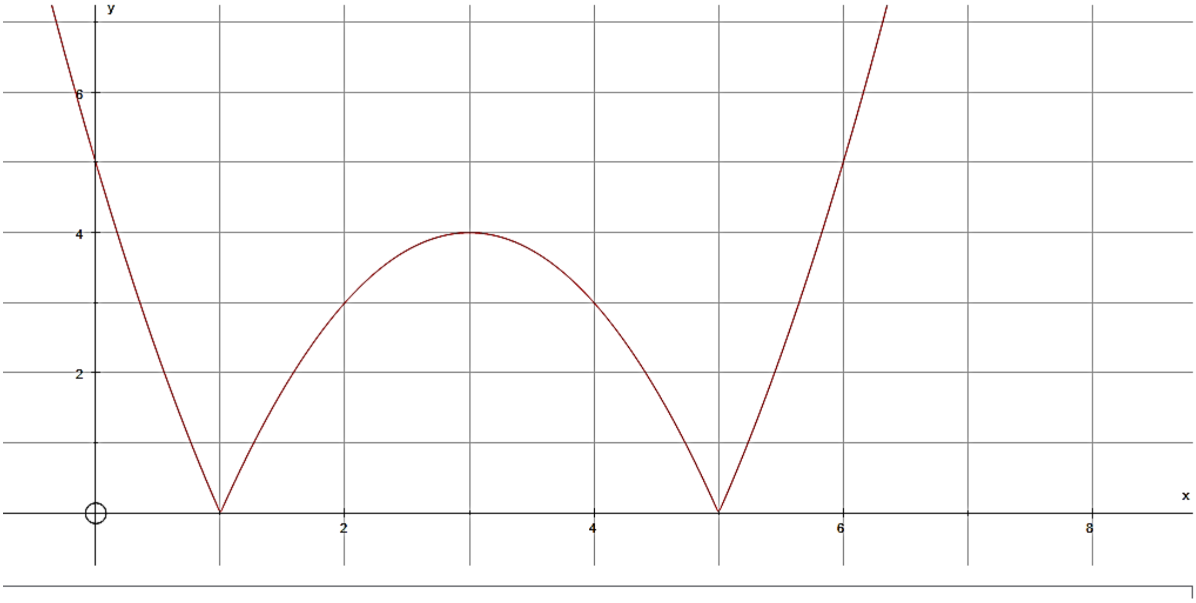
$$p = 2q + 14$$

Gradients equal:

$-2x = q$ $\qquad \lim_{x \to 2^-} f'(x) = \lim_{x \to 2^+} f'(x)$

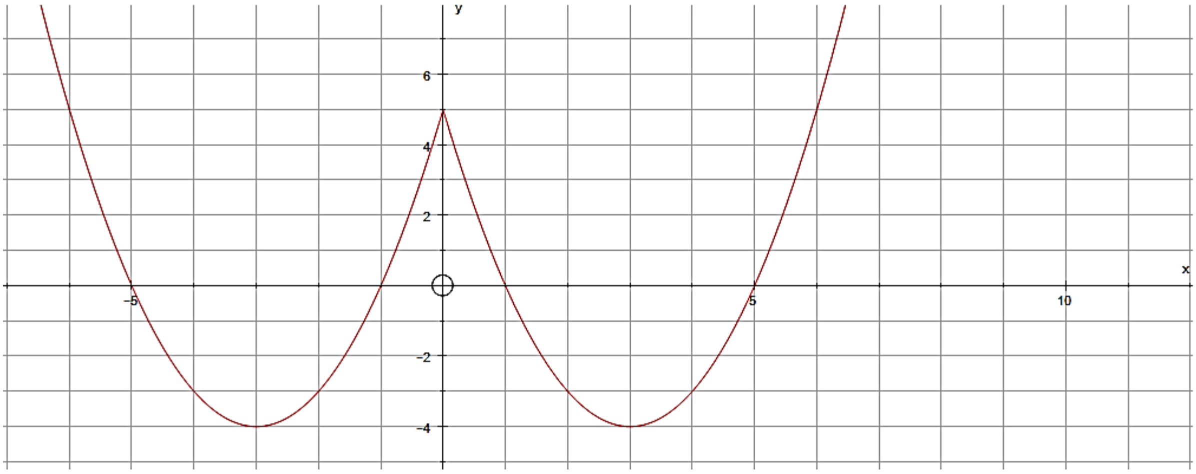
$q = -4$ $\qquad p = 6$ (8)

3.3 (a)



(5)

(b) Shape



(8)

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QUESTION 4

Prove true for n = k + 1

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\cdots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$$

LHS $= \frac{k(k+2)+1}{(k+1)(k+2)}$

$= \frac{(k+1)^2}{(k+1)(k+2)}$

$= \frac{k+1}{k+2}$ = RHS

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QUESTION 5

5.1 Area of triangle AOC $= \frac{1}{2}r\sin\theta\cdot r\cos\theta$

$= \frac{1}{2}r^2\sin\frac{\pi}{6}\cdot\cos\frac{\pi}{6}$

$= \frac{\sqrt{3}r^2}{8}$ (6)

5.2 Area sector AOB $= \frac{1}{2}r^2\left(\frac{\pi}{6}\right)$

$= \frac{\pi r^2}{12}$ (4)

5.3 Required area $= \frac{\pi r^2}{12}-\frac{\sqrt{3}r^2}{8}=\frac{2\pi-3\sqrt{3}}{6}$

$\therefore 2\pi r^2-3\sqrt{3}r^2=8\pi-12\sqrt{3}$

$\therefore r^2=4\ \ i.e.\ r=2$ (6)

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QUESTION 6

6.1 B
6.2 E
6.3 C
6.4 D

[12]

QUESTION 7

$$3x^2 - 4y \cdot \frac{dy}{dx} = -4$$
$$\therefore \frac{dy}{dx} = \frac{3x^2+4}{4y}$$
$$\therefore \frac{dy}{dx} = 4$$
$$\therefore y - 1 = 4(x-2)$$
$$\therefore y = 4x - 7$$

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QUESTION 8

8.1 Since 2,1 is close to a turning point, the gradient of the tangent is shallow and the tangent will hit the *x*-axis past D. The tangent will then tend to D. (3)

8.2 $x \neq -2,\ x \neq 2,\ x \neq 4$ (any two) (2)

8.3 $$x_{r+1} = x_r - \frac{\frac{1}{4}x^4 - \frac{4}{3}x^3 - 2x^2 + 16x - 12}{x^3 - 4x^2 - 4x + 16}$$

$x_0 = 3$
$x_1 = 3,45$
$x_2 = 3,464006$
$x_3 = 3,464102$

(8)
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QUESTION 9

9.1 $$f'(x) = 1 - 8(x+1)^{-3}$$
$$0 = 1 - \frac{8}{(x+1)^3}$$
$$\therefore x + 1 = 2$$
$$\therefore x = 1 \quad \text{and} \quad y = 2$$
$$f''(x) = 24(x+1)^{-4}$$
$$\therefore f''(1) = \frac{3}{2} > 0$$
$$\therefore min$$ (10)

9.2 $f(x) = x + \frac{4}{(x+1)^2}$

As $x \to \pm\infty$ $f(x) \to x$

$\therefore y = x$ (2)

9.3 $\int_{-p}^{p} x + 4(x+1)^{-2}\,dx$

$= \left[\frac{x^2}{2} - 4(x+1)^{-1}\right]$

$= \frac{p^2}{2} - \frac{4}{p+1} - \frac{p^2}{2} + \frac{4}{-p+1}$

$= \frac{8p}{1-p^2}$ (6)

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QUESTION 10

10.1 (a) $\int x + 2x^{-\frac{1}{2}} + x^{-2}\,dx$

$= \frac{x^2}{2} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{-1}}{-1} + C$ (7)

(b) $\frac{1}{2}\int 2\tan^5(2x)\cdot\sec^2(2x)\,dx$

$= \frac{1}{2} \times \frac{\tan^6(2x)}{6} + C$

$= \frac{\tan^6(2x)}{12} + C$ (8)

(c) $\int x.(2-x)^{-\frac{1}{2}}\,dx$

Let u = x *du = 1*

$dv = (2-x)^{-\frac{1}{2}}$ $v = -2(2-x)^{\frac{1}{2}}$

$= x.[-2(2-x)^{\frac{1}{2}}\} - \int -2(2-x)^{\frac{1}{2}}.1\,dx$

$= -2x\sqrt{2-x} - \frac{4(2-x)^{\frac{3}{2}}}{3} + C$ (9)

10.2 a = –1 and b = 2

$f(x) = 2x^2 + 1$ (7)

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QUESTION 11

11. Area = length × breadth $= \frac{x}{x^2+4}$

$\frac{dA}{dx} = \frac{(x^2+4)\cdot 1 - x(2x)}{(x^2+4)^2}$

$0 = x^2 + 4 - 2x^2$

$\therefore x = 2$

$\therefore Area = \frac{2}{4+4} = \frac{1}{4}$

[13]

Total: 200 marks